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# A STOCHASTIC MODEL FOR FORECASTING STORM WATER DRAINAGE PATTERNS

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**Abstract**—A stochastic model has been developed for forecasting storm water drainage patterns. This stochastic model can be extremely useful in the design of urban storm water drainage systems. The model takes into consideration rainfall patterns during the past 50 years for the design of storm water drainage systems that can cope with the hydraulic loadings offered by extreme rainfall events and flood events.

## INTRODUCTION

Urban storm water drainage patterns are highly stochastic in nature, with wide fluctuations. The large-scale variations in storm water drainage patterns can be attributed to the highly stochastic nature of rainfall patterns. Planning of future storm water drainage systems in urban areas requires an adequate understanding of future storm water drainage patterns. Obviously, future storm water drainage patterns have to be reasonably forcast to plan the design of present urban storm water drainage systems.

## MODEL DEVELOPMENT AND APPROACH

The conceptual approach assumes that rainfall patterns are highly stochastic in nature. For this purpose, the maximum and minimum rainfall data for the past 50 years is collected. Rainfall patterns during the following several years are expected to vary between the maximum and minimum values observed during the past several years (Fig. 1). Storm water drainage systems, obviously, have to be designed taking into account the maximum likelihood amount of the rainfall during the next several years. For this purpose, the rainfall events are taken from years $t_0$ to $t_n$.

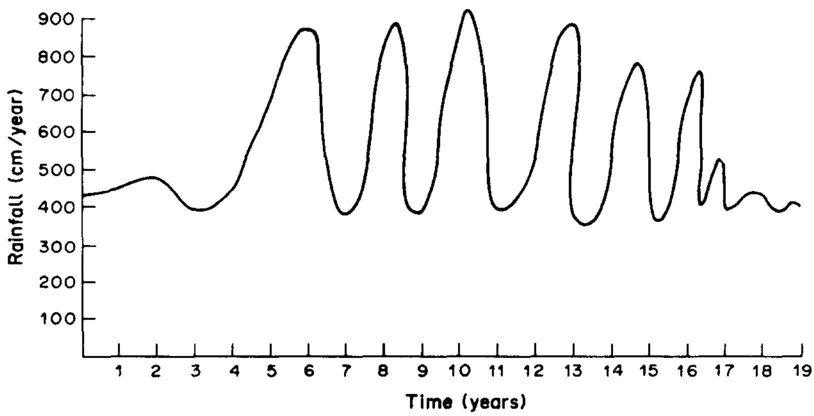


Fig. 1. Diagrammatic representation of the uncertain rainfall patterns.

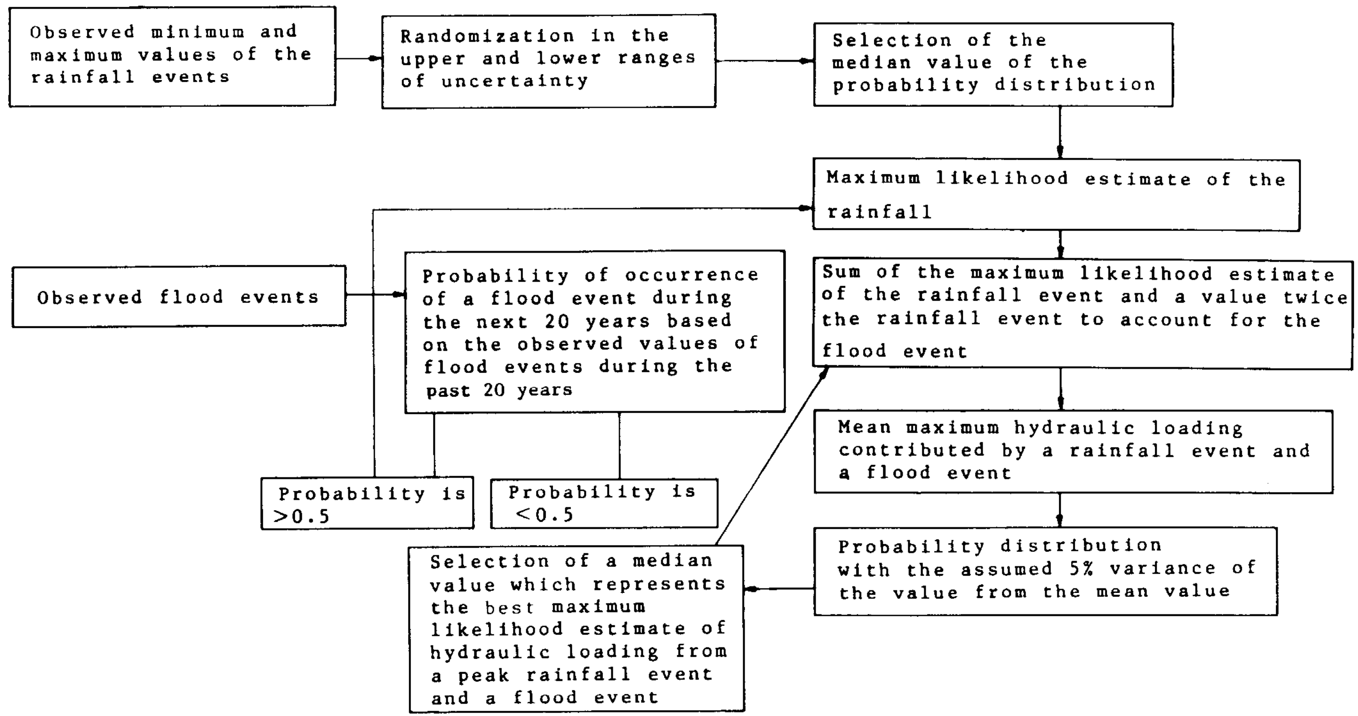


Fig. 2. Flowsheet for the simulation of the best maximum likelihood estimate of the hydraulic loading contributed by a peak rainfall event and a peak flood event to facilitate the design of urban storm water drainage systems.

An average value of the rainfall $(X)_a$ is computed as follows:

$$X_a = \frac{X_0 + \cdots + X_n}{N},$$

where

$X_a$ = average maximum amount of rainfall (cm/day)
$X_0$ = maximum amount of rainfall during the year $t_0$ (cm/day)
$X_n$ = maximum amount of rainfall during the year $t_n$ (cm/day)
and
$N$ = number of years for which the observations have been taken.

To calculate the maximum likelihood value of the rainfall during the following 20 years, a probability distribution is made between the average and the observed maximum value of the rainfall and several random values of the rainfall are simulated between these two ranges. It is reasonable to generate at least 100 random values in this way. The median value of the 100 randomly generated values is a better estimate of the maximum likelihood amount of the rainfall. Storm water drainage system design can then be based on this maximum likelihood value.

The above approach, however, takes into consideration only the rainfall patterns. Storm water drainage patterns can be influenced by sudden storm events such as flooding etc. The possible impact of flooding on storm water drainage patterns has to be taken into consideration when designing storm water drainage systems. The probability of a flood event is computed, based on the available data on flood events during the past several years. In this way, the probability of a flood event during each year of the next 20 years is computed. If a flood event had occurred once in a 20-year period in the past, then the probability of a flood event during each year in the future is 0.05. The flood event is taken into consideration in the design of the storm water drainage system, only if the probability of a flood during each year of the next 20 years is $>0.5$. Otherwise, the flood event is considered highly isolated and is neglected in the design of the storm water drainage system. If the probability of a flood event is $>0.5$, then the possible hydraulic loading contributed by the flood event is taken into consideration when designing the storm water drainage system. For design purposes the net hydraulic loading offered by the flood event can be considered as equivalent to twice the hydraulic loading offered by a peak rainfall event.

The sum of the maximum likelihood estimate of the rainfall event and the hydraulic loading offered by the flood event is considered as the mean value of the total storm water hydraulic loading. A 5% variance from the mean value is then considered and a probability distribution is made between the upper and lower ranges of the uncertainty. The median value of this probability distribution represents the best maximum likelihood estimate of the hydraulic loading, contributed by urban storm water drainage during a year, considering a peak rainfall event and a peak flood event. The design of the storm water drainage system is then based on this value. The conceptual approach has been presented in Fig. 2.

## SUMMARY AND CONCLUSION

A stochastic model has been developed for forecasting storm water drainage patterns. Stochastic modeling of storm water drainage patterns is needed for the design of storm water drainage systems able to cope with the maximum hydraulic loadings offered by both peak rainfall events and flood events. Storm water drainage systems that can cope with the increased hydraulic loadings during the next 20 years of a particular initial period can be designed by utilizing this stochastic model.

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